Fuzzy ideals in $(n+k,n)$-semigroups

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The notion of fuzzy sets, introduced by Zadeh, is a fundamental mathematical concept that deals with uncertainty. It has many applications in a wide range of mathematical areas, as well as in engineering and economics. A fuzzy subset μ of a nonempty set S is a function μ of S into the closed interval $[0,1]$. For all $x \in S$, $\mu(x)$ is called the grade of membership of x. If $\mu(x) = 1$, then we say that x is fully included in S, and, if $\mu(x) = 0$, then we say that x is not included in S. If the set S bears some structure, one may distinguish some fuzzy subsets of S in terms of that additional structure. Fuzzification of algebraic structures was done by Rosenfeld by introducing the notion of fuzzy (left, right) ideal in a groupoid and the notion of fuzzy subgroup of a group.

Motivated by the study of fuzzy ideals in semigroups in some papers of Kuroki, McLean and Kehayopuly, we extend these notions on vector valued semigroups. We present the notion of fuzzy subset of vector valued groupoid (semigroup) $\mathbf{S}$, as well as fuzzy subgroupoid, fuzzy subsemigroup, fuzzy i-ideal (ideal) of vector valued groupoid (semigroup). Suitable examples will be presented as well. Further on, we define prime and fuzzy prime, semiprime and fuzzy semiprime subsets of vector valued groupoids and investigate their properties, as well as properties of fuzzy subsets of vector valued semigroups. We introduce the notions of vector valued bi-ideal and fuzzy bi-ideal on a vector valued semigroup and investigate some of their properties. We characterize the Green's relations $\mathcal{J}_i$ on a vector valued semigroup $\mathbf{S}$ in terms of fuzzy subsets. Green's relations $\mathcal{J}_i^F$ on $\mathbf{S}$ are suitably defined and it is shown that they coincide with the relation $\mathcal{J}_i$ on $\mathbf{S}$.