Interpolation methods on triangular meshes

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A triangular mesh is a 2-dimensional combinatorial manifold or simplicial complex M, whose geometry is defined using a piecewise linear function $f : \Omega \to \mathbb{R}^3$ from a $2d$ domain Ω with the same topology as M.

Given a set of static control points $V_S \subset M$ and a set of moving points $V_M \subset M$, an optimal interpolation is one that minimizes the Dirichlet functional

$$E_D(f) = \int_{\Omega} \|\nabla f\|^2 \, dx.$$

where ∇f is the gradient operator. f also minimizes the total curvature of the resulting triangular mesh. We present two solution methods.

The first method is based on the discrete Laplace equation on a mesh given by

$$\Delta f = 0$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace operator. The second method is known as thin plate spline and is based on using Green functions or radial basis functions to represent f.

Both methods lead to the solution of an over-determined system of (sparse) linear equations in the least squares sense, which are solved using direct or iterative methods. We discuss their implementation in the Math.NET Numerics code library and their computational efficiency. We demostrate applications to 3d graphics including surface deformation, free boundary parametrization and hole closing.

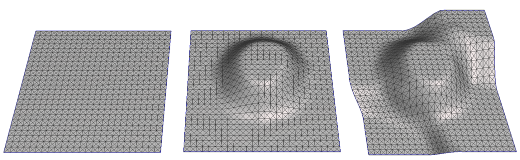

FIGURE 1. Deformation of a plane